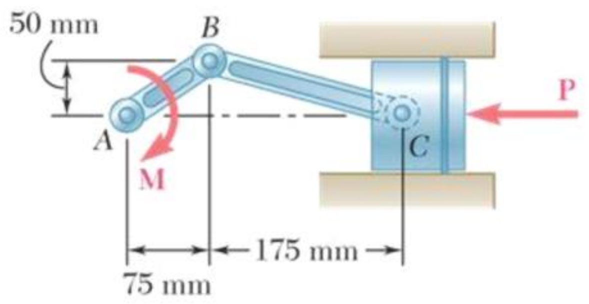
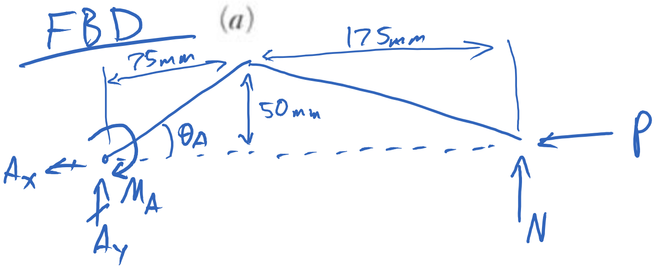
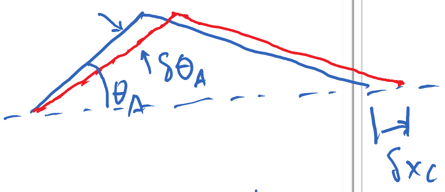


(a)

A couple **M** of magnitude 1.5 kN · m is applied to the crank of the engine system shown. ~~For each of the two positions shown,~~ determine the force **P** required to hold the system in equilibrium.



Virtual Displacements



$\underset{\sim}{M}_A$ is active

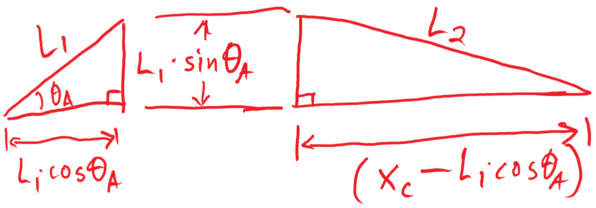
$\underset{\sim}{P}$ is active

$$\delta U = \underset{\sim}{M}_A \cdot \delta\underset{\sim}{\theta}_A + \underset{\sim}{P} \cdot \delta\underset{\sim}{x}_c = 0$$

$$\underset{\sim}{M}_A = -M_A \cdot \hat{k} \qquad \underset{\sim}{\theta}_A = \theta_A \hat{k} \Rightarrow \delta\underset{\sim}{\theta}_A = \delta\theta_A \hat{k}$$

$$\underset{\sim}{P} = -P\hat{i} \qquad \underset{\sim}{x}_c = x_c \hat{i} \Rightarrow \delta\underset{\sim}{x}_c = \delta x_c \hat{i}$$

Write δx_c in terms of $\delta\theta_A$



$$L_2^2 = (L_1 \cdot \sin\theta_A)^2 + (x_c - L_1 \cdot \cos\theta_A)^2$$

solve for x_c

$$X_c = L_1\cdot\cos\theta_A + \sqrt{L_2^2 - (L_1\cdot\sin\theta_A)^2}$$ ← take $\frac{\partial}{\partial\theta_A}$ of this expression for x_c

use chain rule to replace δx_c with $\delta\theta_A$ $\left\{ \delta x_c = \frac{\partial x_c}{\partial\theta_A}\cdot\delta\theta_A \right.$

$$\delta x_c = \overbrace{\left[-\underbrace{L_1\cdot\sin\theta_A}_{50\,mm} - \frac{\overbrace{L_1^2\cdot(\sin\theta_A\cdot\cos\theta_A)}^{(L_1\cdot\sin\theta_A)(L_1\cdot\cos\theta_A)}}{\sqrt{L_2^2 - L_1^2\sin^2\theta_A}}\right]}^{\frac{\partial X_c}{\partial\theta_A}}\cdot\delta\theta_A$$

$$= \left[-(50mm) - \frac{(50mm)(75mm)}{\sqrt{(175mm)^2}}\right]\cdot\delta\theta_A$$

$$= -\left(\frac{500}{7}\right)mm\cdot\delta\theta_A$$

$$\ldots\ \delta U = -M_A\cdot\delta\theta_A - P\cdot\delta x_c = 0$$

$$\Rightarrow\ -M_A\cdot\delta\theta_A - P\cdot\left(-\frac{500}{7}mm\right)\cdot\delta\theta_A = 0$$

$$\underbrace{\left[-M_A - P\cdot\left(-\frac{500}{7}mm\right)\right]}_{\overset{\shortparallel}{0}}\cdot\delta\theta_A = 0$$

$$-M_A + P\cdot\left(\frac{0.5m}{7}\right) = 0$$

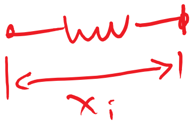
$$\boxed{P = \frac{7\cdot M_A}{0.5m} = \frac{7\cdot(1.5kN\cdot m)}{0.5m} = 21\,kN}$$

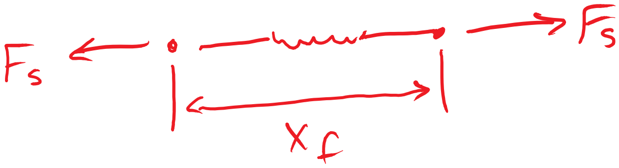
$$\boxed{P = \frac{7 \cdot M_A}{0.5\,m} = \frac{7 \cdot (1.5\,kN \cdot m)}{0.5\,m} = 21\,kN}$$

Torsion Spring

Monday, April 24, 2017 10:56 AM

Linear spring : $F_s = -k \cdot \Delta x$

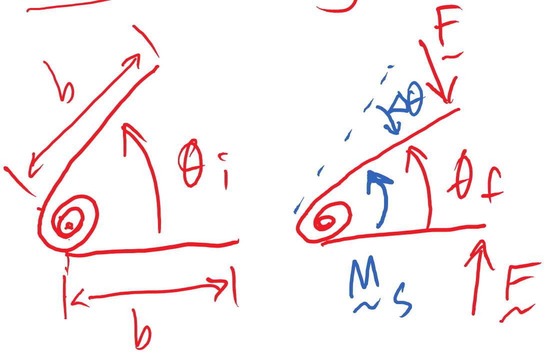




$$\underbrace{F_s}_{\text{force}} = -\underset{\frac{\text{force}}{\text{length}}}{\underset{\uparrow}{k}} \cdot \underbrace{(x_f - x_i)}_{\text{length}}$$

restoring force/moment requires opposite sign for the load and deformation

Torsion Spring



$$\underbrace{\underset{\sim}{M}_s}_{\text{force×length}} = -\underset{\uparrow}{k_s} \underbrace{(\theta_f - \theta_i)}_{\substack{\text{dimensionless} \\ \text{(degrees or radians)}}} \hat{k}$$

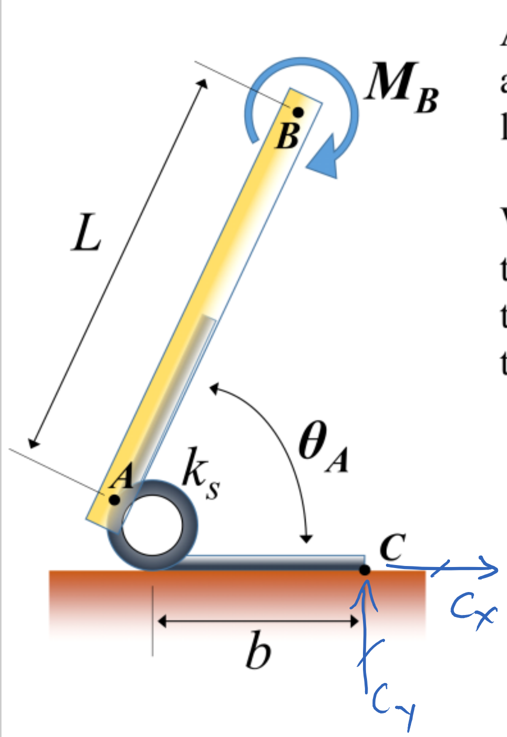
$\underbrace{\text{force×length}}$

$\frac{N\cdot m}{deg.}$ or $\frac{N\cdot m}{rad}$

$\frac{lb\cdot in}{deg}$ $\frac{lb\cdot in}{rad}$

Torsion Spring Example

Tuesday, April 25, 2017 9:32 PM



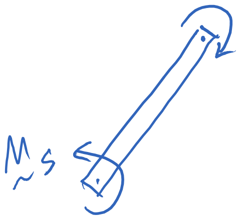
A rod of length L is affixed through a torsion spring at A to a rigid base. The torsion spring has a leg length b and spring constant $k_s = 0.10$ lb-in./degree.

When a moment $M_B = 4$ lb-in. is applied at end B of the rod, as shown, the rod makes an angle $\theta_A = 81°$ to the surface. Find the unloaded angle of the torsion spring.

$\theta_{Af} = 81°$

find θ_{Ai}

$\underset{\sim}{M}_B = -M_B k$



Question

Which terms should be included in the virtual work expression?

[✓ A) $\underset{\sim}{M}_B \cdot \delta\theta_A$] (B) $\underset{\sim}{M}_s \cdot \delta \underset{\sim}{x}_A$

c) $\underset{\sim}{C}_y \cdot \delta \underset{\sim}{r}_c$ (D) $\underset{\sim}{C}_x \cdot \delta \underset{\sim}{r}_A$

$$\delta U = \underset{\sim}{M}_B \cdot \delta \underset{\sim}{\theta}_A + \underset{\sim}{M}_s \cdot \delta \underset{\sim}{\theta}_A = 0$$

$$\underset{\sim}{M}_B = -M_B \hat{k} \qquad \underset{\sim}{M}_s = -k_s \Delta \underset{\sim}{\theta}$$

$$\underset{\sim}{\theta}_A = \theta_A \hat{k} \Rightarrow \delta \underset{\sim}{\theta}_A = \delta\theta_A \cdot \hat{k}$$

$$(-M_B \hat{k}) \cdot (\delta\theta_A \hat{k}) + (-k_s \cdot \Delta \underset{\sim}{\theta}) \cdot (\delta\theta_A \cdot \hat{k}) = 0$$

$$\underbrace{\left[-M_B - k_s(\theta_f - \theta_i)\right]}_{=0}\delta\theta_A = 0$$

$$-k_s(\theta_f - \theta_i) = M_B$$

$$(\theta_f - \theta_i) = -\frac{M_B}{k_s}$$

$$\theta_i = \theta_f + \frac{M_B}{k_s}$$

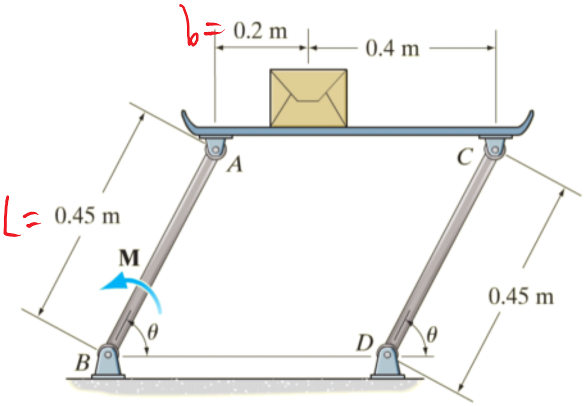
$$= (81^\circ) + \frac{4 \text{ lb-in}}{\left(0.1 \frac{\text{lb-in}}{\text{deg}}\right)}$$

$$= 81^\circ + 40^\circ$$

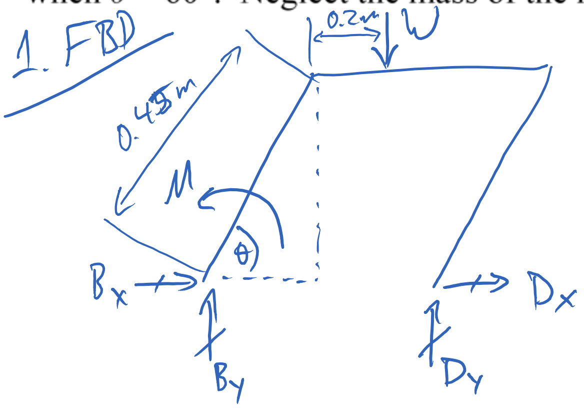
$$\boxed{\theta_i = 121^\circ}$$

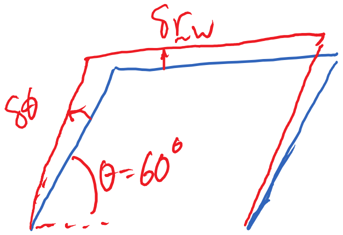
Four-bar linkage example

Tuesday, April 25, 2017 9:32 PM



If the box has a mass of 10 kg, determine the couple moment *M* needed to maintain equilibrium when *θ* = 60°. Neglect the mass of the members.





Active loads

$\underset{\sim}{M}$ & $\underset{\sim}{W}$

Forces, Locations, Virtual Displacements

$\underset{\sim}{M} = M\hat{k} \qquad \underset{\sim}{W} = (-m\cdot g)\cdot\hat{j}$

$\underset{\sim}{\theta} = \theta\hat{k} \Rightarrow \underline{\delta\underset{\sim}{\theta} = \delta\theta\cdot\hat{k}}$

$L = 0.45 m$

$b = 0.2 m$

$\underset{\sim}{r}_w = (L\cdot\cos\theta + b)\hat{i} + (L\cdot\sin\theta)\hat{j}$

$\delta\underset{\sim}{r}_w = \underline{(-L\cdot\sin\theta)\delta\theta\cdot\hat{i}} + \underline{(L\cdot\cos\theta)\cdot\delta\theta\cdot\hat{j}}$

$$\delta U = \underset{\sim}{M}\bullet\delta\underset{\sim}{\theta} + \underset{\sim}{W}\bullet\delta\underset{\sim}{r}_w = 0$$

$$= (M\hat{k})\bullet(\delta\theta\hat{k}) + (-m\cdot g\cdot\hat{j})\bullet\left[\underset{\hat{i}\bullet\hat{j}=0}{\cancel{(-L\cdot\sin\theta\cdot\delta\theta)\hat{i}}} + (L\cdot\cos\theta\cdot\delta\theta)\hat{j}\right] = 0$$

$$= M\cdot\delta\theta - m\cdot g\cdot L\cdot\cos\theta\cdot\delta\theta = 0$$

$$\underbrace{\left[M - m\cdot g\cdot L\cdot \cos\theta\right]}_{=0}\cdot \delta\theta = 0$$

$$M = m\cdot g\cdot L\cdot \cos\theta$$

$$= (10\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(0.45\,\mathrm{m})\cdot \cos(60^\circ)$$

$$\boxed{M = 22.1\ \mathrm{N\cdot m}}$$